Approximations of pseudometrics

Approximation relations on partially ordered sets of pseudometrics on a fixed set X will be considered. An element x_0 of a poset $(X, \leq)$ is said to *approximate from below* an element $x \in X$ (or to be *way below* x, denoted $x_0 \ll x$) if, for any directed set $D \subset X$ such that $x \leq \sup D$, there is an element $d \in D$ such that $x_0 \leq d$. Dually, an element $x_0 \in X$ *approximates* x *from above* an element $x \in X$ (or is *way above* x, denoted $x_0 \gg x$) if, for any filtered set $F \subset X$ such that $\inf F \leq x$, there is an element $f \in F$ such that $f \leq x_0$.

We study these relations on sets

- of all pseudometrics on a set X;
- of all ultrapseudometrics on a set X;
- of all locally compact ultrapseudometrics on a set X;
- of all compact ultrapseudometrics on a set X.

It is shown that only the last set has approximation relations rich enough to provide continuity and dual continuity properties (understood in the sense of domain theory).

Results of joint research with Svyatoslav Nykorovych are presented.