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ONE-POINT EXTENSION PROPRERTY FOR 1-LIPSCHITZ RETRACTIONS DEFINED ON THE UNIVERSAL URYSOHN SPACE $\mathbb{U}$

Let $\mathbb{U}$ be the universal Urysohn space. Let $U\colon \mathbb{U} \to \mathbb{U}$ be a 1-Lipschitz retraction, and define $D\colon \mathbb{U} \to \mathbb{R}$ by

$$D(x) = \operatorname{dist}(x, U[\mathbb{U}]),\ x \in \mathbb{U}.$$

We say that the triple $(\mathbb{U}, U, D)$ has the *one-point extension property (UR)* if the following holds:

For every finite metric space $B = A \cup \{b\}$, every 1-Lipschitz retraction $r\colon B \to B$ such that $r[A] \subseteq A$, every 1-Lipschitz function $p\colon B \to [0, \infty)$, and every isometric embedding $i\colon A \to \mathbb{U}$ satisfying

$$U \circ i(x) = i \circ r(x) \quad \text{and} \quad D \circ i(x) = p(x) \quad \text{for all } x \in A,$$

there exists an isometric embedding $i'\colon B \to \mathbb{U}$ extending i such that

$$U \circ i'(x) = i' \circ r(x) \quad \text{and} \quad D \circ i'(x) = p(x) \quad \text{for all } x \in B.$$

Michal Doucha proved in *Universal and homogeneous structures on the Urysohn and Gurarii spaces*, Israel J. Math. **218** (2017), that such a triple exists. In fact, we may focus solely on U, since $\mathbb{U}$ is fixed and the function D is entirely determined by U. We will show that the set of 1-Lipschitz retractions $U\colon \mathbb{U} \to \mathbb{U}$ satisfying (UR) is a G_δ subset of the set of all $1-$Lipschitz retractions $\mathbb{U} \to \mathbb{U}$ considered with a pointwise topology.

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